**Equations of Motion and Thermodynamics in Numerical Weather Prediction**

For the geostrophic equations of air motion, start with Newton's 2nd Law of Motion:

$$\sum F = ma$$

$$a = \frac{\sum F}{m} \qquad (solve\ for\ the\ acceleration\ of\ the\ air)$$

$$a = \frac{d\boldsymbol{v}}{dt} = \frac{\sum F}{m} = PGF + Coriolis + Gravity + Friction$$

where,

$\boldsymbol{v}$ = the instantaneous velocity of an air parcel (the wind),
$a$ = the acceleration of that air parcel as the wind direction shifts or the speed changes,
$t$ = time,
$\frac{\sum F}{m}$ = the sum of the forces per unit mass (think of "unit mass" as a parcel or "block" of air).

The forces upon a parcel of air are summed to measure the total acceleration of the air (wind). The two "biggies" are the pressure gradient force (*PGF*) and the Coriolis force. The *PGF* makes the air flow from high air pressure to low air pressure, and the Coriolis force mathematically explains the rotation of the earth. Gravity and friction also play a role, but the magnitudes of these forces are lower. To visualize the *PGF*, think of a change in air pressure across a fixed area (i.e., one-dimensionally, think of the distance from DC to Baltimore). As the pressure gradient increases, the wind speed increases. A large difference in pressure between Washington, DC, and Baltimore results in a windy drive along the Baltimore-Washington Parkway. The rotation of the earth is a constant, and it turns the wind to the right (left) in the Northern Hemisphere (Southern Hemisphere). This makes the air flow in a circle around high and low pressure systems.

Think of the wind or air velocity as a vector in three dimensions, $x$, $y$, and $z$. $x$ represents the east-west axis, $y$ represents the north-south axis, and $z$ represents vertical motion. In the vertical, the total momentum is primarily driven by the two largest of several forces: gravity and the vertical pressure gradient, known to meteorologists as the hydrostatic equation. Vertical motion is very important in meteorology. Rising air expands and cools, and this eventually leads to saturation, clouds, and precipitation. Sinking air compresses and warms, and this creates a drying affect. With a single equation for each dimension of the wind, we are up to three equations. The following physical laws or equations bring us to a total of six:

- **The Continuity Equation**, which represents law of the conservation of mass—mass cannot be created or destroyed. Once a parcel of air moves, it must be replaced by a neighboring parcel. The transport of air or water will not result in a vacuum.
- The **Thermodynamic Energy Equation** comes from the First Law of Thermodynamics, which represents the conservation of energy, which cannot be created or destroyed. Recall,

$Kinetic\ Energy = \frac{1}{2}m\boldsymbol{v}^2$. Sometimes, kinetic energy is converted to potential energy, and vice versa. Definitions: $m$ is the mass, and the vector $\boldsymbol{v}$ is the wind.

- The **Ideal Gas Law**: $p = \rho RT$ This is often taught in high school chemistry and correlates the density of a gas or a liquid to its temperature or pressure. For example, if you increase the temperature of a gas at a constant density (or volume), the pressure will increase. Definitions: $p$ is pressure, $T$ is temperature, $R$ is the ideal gas constant, and $\rho$ is density.

The above represent the six "dry" primitive equations, and collectively, they all contain six unknowns. Recall from algebra, if you have six equations and six unknowns, you can solve for each of the unknowns. The "unknowns" are the wind (three unknowns, one for each dimension, x, y, and z), pressure, density, and temperature. A seventh equation that represents the conservation of water vapor (a seventh unknown) is required to fully describe a moist atmosphere.

Several, but not all of the above, are differential equations with respect to time. This means they can be integrated forward in time, making them predictive equations. They are initialized with observed weather conditions (observed temperature, pressure, wind, density, and water vapor) two or four times per day, depending upon the model. Then, the equations are integrated forward in time to get a picture of future weather, out to about two weeks. Most predictive forecast skill drops to near zero after seven days, but similar models are used for climate.

Here's an application of continuity: **Upwelling.** Along the East coast in the summer during a heat wave, the surface wind direction is often from the south or southwest. The rotation of the earth turns the upper-most meters of the water about 90 degrees to the right (in the clockwise direction). This results in surface water moving away from the shoreline, toward the east or southeast. With all that surface water moving out to sea, water must replace it, and the only source for replacement is water from below. Water at the bottom of the ocean is always cold so cold water moves upward to the surface. Suddenly in the middle of a summer heat wave, the sea surface temperature may drop 10 degrees or more—from the low- or mid-70s to the upper-50s, which is bone-chilling. It is strange to see some of the chilliest sea surface temperatures of the summer suddenly develop during a prolonged heat wave. These cold waters usually don't last very long. Once the wind shifts, the ocean currents shift, the cold water disperses, and the warmer water temperatures are restored.

**Down-welling** is the opposite condition. North or northeast winds, impacting the surface water current, turn the mass transport of the upper-most meters of the Atlantic Ocean to the right. This brings warmer surface water, sometimes affected by the Gulf Stream, westward or northwestward toward the coastline, warming the sea surface temperature. With a northeast wind, it is probably a cloudy, chilly, and it may be raining, but the water temperature is balmy. Don't attempt to go in or near the water if there is any hint or danger of thunder!